Thin Film Interference

AP Physics

[images courtesy of Andrew Duffy]
Thin Film Interference

Interference between light waves is the reason that thin films, such as soap bubbles, show colorful patterns. This is known as thin-film interference; interference between light waves reflecting off the top surface of a film with waves reflecting from the bottom surface. To obtain a nice colored pattern, the thickness of the film has to be comparable to the wavelength of light.
Thin Films: What is happening

We can break the process into 4 steps.

1: A light wave is incident on the top surface of the film.
Thin Films: What is happening

We can break the process into 4 steps.

2: The incident wave is partly reflected (possibly inverting) and partly transmitted. The reflected wave is moved to the right so we can see it.
Thin Films: What is happening

We can break the process into 4 steps.

3: The transmitted wave also partly reflects off the bottom surface of the film (possibly inverting). This second reflected wave is moved to the far right.
Thin Films: What is happening

We can break the process into 4 steps.

4: The two reflected waves interfere with one another as they exit the film.

The film thickness needs to be just right if we want completely constructive or completely destructive interference.

http://physics.bu.edu/~duffy/semester2/c26_thinfilm.html
Thin Films: What is happening

What kind of interference do we have here?

In this case, the film thickness is exactly one wavelength, so the wave that reflects off the bottom surface of the film travels a down-and-back extra distance of 2 wavelengths compared to the wave reflecting off the top surface.

A: Constructive
B: Destructive
Thin Films: What is happening

What kind of interference do we have here?

Even though the extra distance traveled is an integer number of wavelengths, we can see that the reflected waves interfere destructively. This is because the wave reflecting off the bottom surface is inverted, which is like an extra half-wavelength shift.

A: Constructive
B: Destructive
Thin Films: 5 step approach

Let’s use a five-step method to analyze thin films. The basic idea is to determine the effective path-length difference ($\Delta$) between the wave reflecting from the top surface of the film and the wave reflecting from the bottom surface.

The effective path-length difference accounts for the extra distance of $2t$ traveled by the wave that reflects from the bottom surface, as well as any inversions. Also we must realize that we are talking about the light in the material, so we must use the wavelength of the light in the film, $\lambda_{\text{film}}$.
Step 1: $\Delta_{\text{top}}$

If the wave goes from a less dense to more dense media, the reflected wave is inverted. If it goes from more dense to less dense, it does not.

So,

\[ \text{If } n_2 > n_1, \quad \Delta_{\text{top}} = \frac{\lambda_{\text{film}}}{2} \]

\[ \text{If } n_2 < n_1, \quad \Delta_{\text{top}} = 0 \]
Step 2: $\Delta_{\text{bottom}}$

The second reflected wave travels an additional $2t$ (2 thicknesses) and can be inverted. So we have at least $2t$.

So, if $n_3 > n_2$, \[ \Delta_{\text{bottom}} = 2t + \frac{\lambda_{\text{film}}}{2} \]

If $n_3 < n_2$, \[ \Delta_{\text{bottom}} = 2t \]
Step 3: $\Delta_{\text{total}}$

The effective path-length difference is now found by subtracting the top shift from the bottom shift.

$$\Delta = \Delta_{\text{bottom}} - \Delta_{\text{top}}$$
Step 4: Apply Interference Condition

If you remember from before, the path length difference must be related to the wavelength in a certain way for constructive or destructive interference to occur.

For constructive interference
\[ \Delta = m \lambda_{film} \]

For destructive interference
\[ \Delta = \left( m + \frac{1}{2} \right) \lambda_{film} \]
Step 5: Solve the equation

Now all we have to do is solve the resulting equation.

Remember there is a solution for different values of m (0, 1, 2, ...). We are usually asked for the smallest wavelength or thickness, so we generally will solve for m=0 or m=1, depending on which one gives a nonzero answer.

And remember,

$$\lambda_{film} = \frac{\lambda_{vacuum}}{n_{film}}$$
An Example

White light in air shines on an oil film of thickness $t$ that floats on water. The oil has an index of refraction of 1.50, while the refractive index of water is 1.33.

When looking straight down at the film, the reflected light looks orange, because the film thickness is just right to produce completely constructive interference for a wavelength, in air, of 600 nm.

What is the minimum possible thickness of the film?
Step 1

Determine $\Delta_{top}$, the shift for the wave reflecting from the top surface of the film.

\[ A: \quad \Delta_{top} = \frac{\lambda_{film}}{2} \quad n_{air} < n_{oil} \]

\[ B: \quad \Delta_{top} = 0 \]
Step 2

Determine $\Delta_{\text{bottom}}$, the shift for the wave reflecting from the bottom surface of the film.

$A: \quad \Delta_{\text{bottom}} = 2t + \frac{\lambda_{\text{film}}}{2}$

$B: \quad \Delta_{\text{bottom}} = 2t$

$n_{\text{oil}} > n_{\text{water}}$
Step 3

Determine $\Delta$, the total effective path length difference for the two reflected waves.

\[ A : \quad \Delta = 2t + \frac{\lambda_{film}}{2} \]

\[ B : \quad \Delta = 2t \]

\[ C : \quad \Delta = 2t - \frac{\lambda_{film}}{2} \]
Step 4

Determine the appropriate interference condition.

Constructive

\[ A: \quad 2t - \frac{\lambda_{film}}{2} = m\lambda_{film} \]

\[ B: \quad 2t - \frac{\lambda_{film}}{2} = (m + \frac{1}{2})\lambda_{film} \]
Step 5: Let’s Solve It

$n_{film} = 1.5, \lambda = 600 \text{ nm}, m = ?$

\[ 2t - \frac{\lambda_{film}}{2} = m \lambda_{film} \]

A: $t_{\text{min}} = 100 \text{ nm}$

B: $t_{\text{min}} = 150 \text{ nm}$

C: $t_{\text{min}} = 300 \text{ nm}$

D: $t_{\text{min}} = 450 \text{ nm}$
Step 5: Let’s Solve It

\( n_{\text{film}} = 1.5, \, \lambda = 600 \, \text{nm}, \, m = 0 \)

\[
\lambda_{\text{film}} = \frac{\lambda}{n} = \frac{600\,\text{nm}}{1.5} = 400\,\text{nm}
\]

\[
2t - \frac{\lambda_{\text{film}}}{2} = m\lambda_{\text{film}} = 0
\]

\[
2t = \frac{\lambda_{\text{film}}}{2} \Rightarrow t = \frac{\lambda_{\text{film}}}{4} = \frac{400\,\text{nm}}{4} = 100\,\text{nm}
\]
One last question?

Why do we get horizontal bands on the soap film?

Gravity causes the film to be thicker at the bottom, with decreasing thickness as you move up. Different thicknesses correspond to different colors that fit will undergo constructive interference. Each repetition represents another $m$ that works with that color.